

Solving coupled Dyson–Schwinger equations on a compact manifold

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Abstract

We present results for the gluon and ghost propagators in $SU(N)$ Yang–Mills theory on a four-torus at zero and nonzero temperatures from a truncated set of Dyson–Schwinger equations. When compared to continuum solutions at zero temperature sizeable modifications due to the finite volume of the manifold, especially in the infrared, are found. Effects due to non-vanishing temperatures T , on the other hand, are minute for $T < 250$ MeV.

Key words: Nonlinear integral equations, Infrared behaviour, Non-perturbative QCD, Running coupling, Dyson–Schwinger equations
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1 Introduction

The infrared behaviour of QCD Green’s functions is intimately related to the issue of confinement, see *e.g.* ref. [1]. The confinement phenomenon is intrinsically non-perturbative, and thus a quite limited number of methods is suited to investigate this and related issues. Furthermore, the infrared singular nature of some of the QCD Green’s functions provides a further severe obstacle in the study of these quantities. In ref. [2] a numerical method is described where the analytical treatment of the gluon and ghost propagator Dyson–Schwinger equations (DSEs) for momenta smaller than a certain matching

scale allowed for the numerical solution of these coupled non-linear integral equations. At this point the question arises whether a purely numerical method to solve these equations alleviates their treatment. To this end one notes that on a compact manifold the finite volume of underlying space-time acts as a natural infrared regulator. Using a four-torus this will, on the one hand, be exploited here for a direct solution of the coupled gluon and ghost DSEs, and, on the other hand, be utilized to estimate effects of the finite volume. An extension to asymmetric tori is equivalent to the choice of non-vanishing temperatures, and thus it will also be considered in this paper.

As noted above confinement manifests itself in the long range behaviour of Yang–Mills theory. According to the Kugo–Ojima confinement criterion [3,4] as well as the Gribov–Zwanziger horizon scenario, see *e.g.* ref. [5], it is expected that the gauge fixing degrees of freedom and thus the Faddeev–Popov ghosts provide the long-range correlations. In Landau gauge this has been confirmed in the framework of DSEs as well as exact renormalisation group equations in a series of papers [5,6,7,8,9,10,11,12]: the ghost propagator is indeed more singular in the infrared than a simple pole. The gluon propagator, on the other hand, vanishes in the infrared. Lattice Monte-Carlo calculations agree with these findings, although it is currently under debate whether the lattice gluon propagator is finite [13] or vanishes in the infrared [14]. Here a study of DSEs at finite volumes can be helpful when judging the finite-volume effects in lattice calculations. As due to the periodic boundary conditions used in lattice calculations the underlying space-time manifolds are effectively four-tori, we will employ these manifolds in our study. In this paper we will present general techniques needed to solve the related DSEs, and we will comment on the differences in the method and for the solutions when compared to the treatment based on flat Euclidean space-time [2,7,15,16].

The paper is organised as follows: In the next section we summarise general properties of DSEs on a compact manifold. We discuss the boundary conditions of the fields, present the equations for finite temperature and discuss the renormalisation. Numerical results are presented in section 3: Finite volume effects are discussed in subsection 3.1, and results for non-vanishing temperatures are given in section 3.2. We conclude with a summary and an outlook on further applications in section 4. The employed integral kernels are listed in appendix A and details of the numerical method are presented in appendix B.

2 Dyson-Schwinger equations on a compact manifold

The DSEs are equations for the Green’s functions, *i.e.* for the vacuum expectation values of time-ordered products of quantum fields. Therefore it is

quite illustrative to consider first the general properties of quantum fields on compact space-time manifolds.

2.1 Quantum fields on a compact Euclidean space-time manifold

Quantum fields on a compact Euclidean space-time manifold are realised by imposing (anti-)periodic boundary conditions:

$$\Phi(x^\mu + L^\mu) = e^{i2\pi\eta}\Phi(x^\mu) . \quad (1)$$

Here Φ is shorthand for all fields that appear in the Lagrangian of the theory, and L^μ denotes the length of the torus in the μ -th direction. The overall phase is $\eta = 0$ for bosons, *i.e.* periodic boundary conditions, and $\eta = \frac{1}{2}$ for fermions, *i.e.* antiperiodic boundary conditions. Furthermore, in d -dimensional coordinate space $\Phi(x)$ has the following Fourier representation

$$\Phi(x) = \frac{1}{L_1 \cdots L_d} \sum_{n_1 \dots n_d} \Phi_{n_1 \dots n_d} e^{i\omega_\mu x_\mu} , \quad (2)$$

$$\omega_\mu = \frac{2\pi}{L_\mu}(n_\mu + \eta) , \quad (3)$$

in terms of its Fourier components $\Phi_{n_1 \dots n_d}$. The ω_μ are discretized momenta, and due to an obvious analogy we will call these Matsubara momenta or Matsubara frequencies. The special case we want to study here is $d = 4$, *i.e.* three space dimensions ($\mu = 1, 2, 3$) and one ‘time’ ($\mu = 4$) dimension.

In the framework of the imaginary time formalism quantum field theory at non-vanishing temperatures is related to the corresponding Euclidean path-integral with compactified time-direction such that $\Delta\tau = \beta = 1/T$. Therefore torus-compactified Euclidean space-time with a shorter length for the time direction introduces an effective temperature which can be estimated to be

$$T = 1/L_4 \text{ with } L_4 \ll L . \quad (4)$$

Depending on the application we will either set all L_μ equal, or we choose $L_1 = L_2 = L_3 = L$ and $L_4 = \beta = 1/T$.

For flat Euclidean space-time the DSEs in momentum space are integral equations. Due to the representation of the fields in Fourier space of compact space-time we deal in this case with sums over Matsubara momenta, *i.e.* when compactifying space-time to a torus the DSE integrals are replaced by sums over the n_μ :

$$\int \frac{d^4 q}{(2\pi)^4} (\cdots) \longrightarrow \frac{T}{L^3} \sum_{n_4} \sum_{n_1, n_2, n_3} (\cdots) . \quad (5)$$

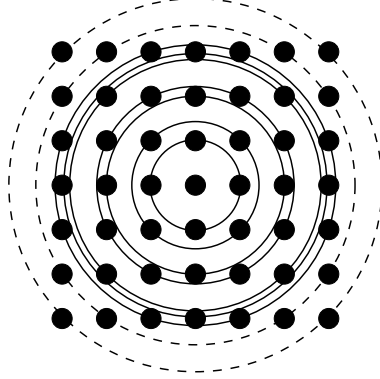


Fig. 1. Sketch of the momentum grid dual to the four-torus. The summation over complete hyperspheres is indicated by fully drawn circles. The hyperspheres depicted by dashed lines are not complete if one uses Cartesian cutoffs instead of an $O(3)$ invariant one. Note that an $O(4)$ -invariant cutoff can only be chosen in the zero temperature case.

For the numerical treatment of the equations it is convenient to rearrange this summation such that they represent a spherical coordinate system [17], see fig. 1 for an illustration. Employing the $O(3)$ symmetry of the space directions we write

$$\frac{T}{L^3} \sum_{n_4} \sum_{n_1, n_2, n_3} (\cdots) = \frac{T}{L^3} \sum_n \sum_{j, m} (\cdots) , \quad (6)$$

with $n \equiv n_4$. Furthermore, j denotes spheres with $n_i n_i = \text{const}$, $i = 1 \dots 3$, and m numbers the grid points on a given sphere. This spherical summation of Cartesian grid points is very useful device when introducing $O(3)$ invariant cut-offs.

On a torus the gluon as a boson clearly obeys periodic boundary conditions. The ghost field occurs when representing the Faddeev–Popov determinant as a Grassmannian functional integral. Although the ghost fields c and \bar{c} are thus anticommuting fields they also have to obey periodic boundary conditions. Including fermionic fields Ψ in the fundamental representation this can be seen easily from the ghost fields’ properties under BRST-transformations:

$$s\Psi = -igt^a c^a \Psi . \quad (7)$$

Here the t^a are the generators of $SU(N)$, and s is the Slavnov operator. Since s is continuous and thus unique, it is periodic. To preserve the antiperiodicity (due to the fermionic nature of the matter field) on both sides of the equation periodic boundary conditions for the ghost field c are required.

2.2 The Dyson-Schwinger Equations

The fully renormalized zero temperature ghost and gluon propagators in Landau gauge can always be written in the form

$$D_G(k) = -\frac{G(k^2)}{k^2}, \quad (8)$$

$$D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \frac{Z(k^2)}{k^2}, \quad (9)$$

where $G(k^2)$ and $Z(k^2)$ denote the ghost and gluon dressing functions, respectively. A combination of these functions can be used to define the nonperturbative running coupling [6]

$$\alpha(k^2) = \alpha(\mu^2) G^2(k^2, \mu^2) Z(k^2, \mu^2). \quad (10)$$

which is a renormalization group invariant, *i.e.* independent of the renormalization scale μ .

For non-vanishing temperatures, in the rest frame of the heat-bath, one has two different tensor structures in the gluon propagator, one transverse and one longitudinal to the heat bath [18,19]:

$$D_{\mu\nu}^{ab}(k) = \frac{\delta^{ab}}{k^2} \left(P_{\mu\nu}^T(k) Z_m(k_0, |\vec{k}|) + P_{\mu\nu}^L(k) Z_0(k_0, |\vec{k}|) \right) \quad \text{with} \quad (11)$$

$$P_{ij}^T(k) = \delta_{ij} - \frac{k_i k_j}{k^2}, \quad P_{00}^T = P_{i0}^T = P_{0i}^T = 0,$$

$$P_{\mu\nu}^L(k) = P_{\mu\nu}(k) - P_{\mu\nu}^T(k), \quad P_{\mu\nu} = \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}, \quad (12)$$

$$i, j = 1, 2, 3; \quad \mu, \nu = 1, 2, 3, 4,$$

and $k^2 = k_0^2 + \vec{k}^2$. The ghost propagator is a Lorentz scalar and does not acquire further structures but its dressing function will depend then separately on frequency and three-momentum:

$$D_G^{ab}(k) = -\frac{\delta^{ab}}{k^2} G(k_0, |\vec{k}|). \quad (13)$$

The following linear combination of gluon dressing functions,

$$Z_T(k) := \frac{1}{3} Z_0(k) + \frac{2}{3} Z_m(k), \quad (14)$$

will prove to be closest to the zero temperature gluon dressing function.

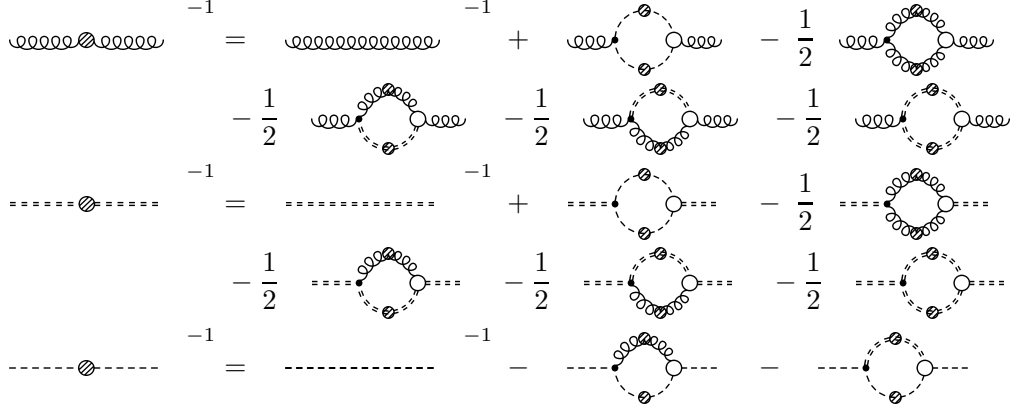


Fig. 2. Diagrammatic representation of the propagator DSEs in the truncation scheme used in this work. Wiggly lines denote heat-bath transverse gluon propagators, double-dashed lines are heat-bath longitudinal propagators and dashed lines represent ghost propagators. Blobs indicate dressed propagators and vertex functions, respectively.

The diagrammatic structure of the Dyson-Schwinger equations for the ghost and gluon propagators is shown in fig. 2. It is a coupled, nonlinear system of equations, which contains dressed propagators as well as dressed vertex functions on the right hand side. In order to obtain a closed system of equations it is necessary to specify suitable approximations for these vertices. Such a truncation scheme has been developed and successfully applied in ref. [17]. As the scheme is discussed in great detail in ref. [15], we only mention briefly its main ideas here. The key observation is ghost dominance in the infrared [12], *i.e.* the ghost loop in the gluon DSE dominates all contributions from the gluon diagrams for small momenta. It is furthermore known, that the bare ghost-gluon vertex,

$$\Gamma_{\mu}^{gh-gl}(p, q) = iq_{\mu} \quad (15)$$

is an excellent approximation in both the infrared and ultraviolet momentum regime [20,21]. Here q_{μ} is the momentum of the outgoing ghost-leg and the colour-indices have been suppressed. The gluon two-loop diagrams, involving the full four-gluon vertex, can be neglected in these regions and it turned out to be a suitable approximation to neglect contributions from the four-gluon vertex altogether. For the three-gluon vertex a minimal dressing has been chosen such that the correct perturbative anomalous dimensions of the propagators in the ultraviolet are recovered in the solutions. The vertex is given by

$$\Gamma_{\mu\nu\lambda}^{3g}(k, p, q) = \Gamma_{\mu\nu\lambda}^{3g,0}(k, p, q) H_{3g}(k, p, q) \quad (16)$$

where $\Gamma_{\mu\nu\lambda}^{3g,0}(k, p, q)$ is the bare vertex. The dressing function H_{3g} is given by

$$H_{3g}(k, p, q) = \frac{1}{Z_1} \frac{G(q)^{(-2-6\delta)}}{Z(q)^{(1+3\delta)}} \frac{G(p)^{(-2-6\delta)}}{Z(p)^{(1+3\delta)}}, \quad (17)$$

with the momenta q and p running inside the loop. Here the anomalous dimen-

sion of the ghost propagator, $\delta = -9/44$, together with the vertex renormalization constant Z_1 ensure the correct ultraviolet running of the vertex with momenta and renormalization scale. Furthermore this choice leads to cutoff independent ghost and gluon dressing functions in the continuum. This truncation scheme has been generalised to non-vanishing temperatures in ref. [22,23,24].

On an asymmetric torus this leads to the following set of DSEs for the ghost and gluon dressing function $G(k)$, $Z_m(k)$ and $Z_0(k)$:

$$\frac{1}{G(k)} = \tilde{Z}_3 + g^2 N_C \tilde{Z}_1 \frac{T}{L^3} \sum_{n,j,m} \frac{G(q)}{k^2 q^2 (k-q)^2} (A_T Z_m(k-q) + A_L Z_0(k-q)) , \quad (18)$$

$$\begin{aligned} \frac{1}{Z_m(k)} = Z_3 - \frac{1}{2} g^2 N_C \tilde{Z}_1 \frac{T}{L^3} \sum_{n,m,j} \frac{G(q)G(p)}{k^2 q^2 p^2} R + \frac{1}{2} g^2 N_C Z_1 \frac{T}{L^3} \sum_{n,m,j} \frac{H_{3g}(q,p,k)}{k^2 q^2 p^2} \\ (M_T Z_m(q) Z_m(p) + M_1 Z_0(q) Z_m(p) + M_2 Z_0(p) Z_m(q) + M_L Z_0(q) Z_0(p)) , \end{aligned} \quad (19)$$

$$\begin{aligned} \frac{1}{Z_0(k)} = Z_3 - g^2 N_C \tilde{Z}_1 \frac{T}{L^3} \sum_{n,m,j} \frac{G(q)G(p)}{k^2 q^2 p^2} P + g^2 N_C Z_1 \frac{T}{L^3} \sum_{n,m,j} \frac{H_{3g}(q,p,k)}{k^2 q^2 p^2} \\ (N_T Z_m(q) Z_m(p) + N_1 Z_0(q) Z_m(p) + N_2 Z_0(p) Z_m(q) + N_L Z_0(q) Z_0(p)) . \end{aligned} \quad (20)$$

The expressions for the kernel functions A_T , A_L , R , M_T , M_1 , M_2 , M_L , P and N_T , N_1 , N_2 , N_L are given in appendix A, see eqs. (A.1)-(A.12). The corresponding O(4)-symmetric version of these equations in the zero temperature limit can be found in ref. [17].

2.3 Renormalisation

We employ a momentum subtraction (MOM) scheme to renormalise the DSEs on the torus. For zero temperature, *i.e.* on a symmetric torus, this scheme has been described in [17]. The DSEs for the (renormalised) dressing functions $D \in \{G, Z_m, Z_0\}$ at the four momentum k^2 can be written symbolically as

$$\frac{1}{D(k_i^2)} = Z^D + \Pi_D(k_i^2) , \quad (21)$$

where Z^D stands for the respective renormalisation constants $\tilde{Z}_3, Z_{3m}, Z_{30}$ and $\Pi_D(k^2)$ denote the loop contributions. The momenta k_i^2 are lying on the hyperspheres shown in fig. 1. To eliminate the momentum independent renormalisation factors Z^D we subtract eqs. (21) from their respective equations evaluated at a fixed subtraction scale s . This scale is conveniently chosen equal

to the squared renormalisation point, $s = \mu^2$. We obtain:

$$\frac{1}{D(k^2)} = \frac{1}{D(\mu^2)} + \Pi_D(k^2) - \Pi_D(\mu^2) . \quad (22)$$

One then has to specify a renormalisation condition, *i.e.* a value for $D(\mu^2)$, to complete the (re-)normalisation procedure. For convenience this condition has been adapted from the continuum solution, given in ref. [10]. Using $\alpha(\mu^2) = 0.968$ the renormalisation point is given by $\mu^2 = 1.610 \text{ GeV}^2$ and the normalisation condition is $Z(\mu^2) = 0.847$. The corresponding value of the ghost dressing function is then $G(\mu^2) = 1/\sqrt{Z(\mu^2)} = 1.087$, *c.f.* eq. (10).

On the asymmetric torus we exploit the fact that no new divergences can arise at finite temperatures in a renormalisable quantum field theory [18,19]. Only the finite part of the renormalisation, *i.e.* the condition on $D(s)$, can be affected. If the subtraction scale s is much larger than $\max\{\Lambda_{\text{QCD}}, 1/T\}$, where the dressing functions are not affected by temperature effects, it should suffice to determine only one renormalisation constant for all Matsubara frequencies of each respective propagator. These constants are determined *e.g.* from the zeroth order term in the Matsubara sum ($n = 0$) by the condition

$$Z^D = \frac{1}{D(0, s)} - \Pi_D(0, s) , \quad (23)$$

where $D(0, s)$ can be conveniently set to the zero temperature value $D(s)$. We checked that within a very small error we obtain the same value for the constant Z^D also for all other Matsubara frequencies. The overall temperature dependence of the renormalisation constants is several percent for temperatures in the range from 50 MeV to 800 MeV. The renormalisation constants for the heat-bath transverse and longitudinal gluon dressing functions differ slightly, but at most a few percent for increasing temperatures.

In regularisation schemes with an ultraviolet cutoff one also has to deal with quadratic divergencies, which cannot be accounted for by the MOM-procedure described above. In the continuum formulation of the DSEs quadratic divergencies are removed on the level of the DSEs by a careful procedure [10,22] which amounts to adding a suitable counterterm to the bare action of the theory. A similar method has been applied to the formulation on the symmetric and asymmetric torus and is described in detail in [17,23]. We therefore refrain from repeating the details here and just note that the integral kernels given in appendix A contain kernels where the dangerous terms have already been eliminated.

The dressing functions are calculated on a momentum grid that is limited by the ultraviolet cut-off Λ . However, as can be seen directly in equations (18)–(20), the integral kernels and the dressing functions in the loops depend

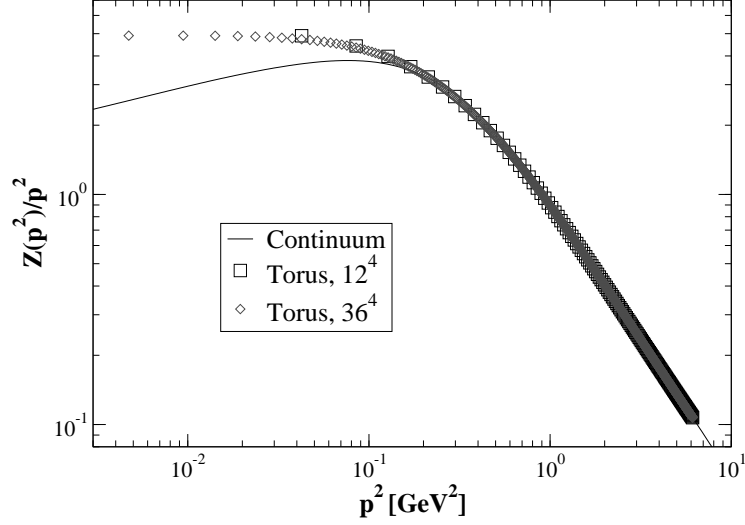


Fig. 3. The gluon propagator $Z(p^2)/p^2$ in the continuum and on the torus at two different volumes.

$\Lambda[\text{GeV}]$	2.47	3.09	3.71	4.33	4.95
$D(0)[\text{GeV}^{-2}]$	4.90	4.55	4.23	3.97	3.75

Table 1

Finite size analysis: Dependence of $D(0)$ on the cutoff Λ at a fixed volume of $V = 4091 \text{ fm}^4$.

on the relative momentum between the momentum k from outside the loop and the loop momentum q . These relative momenta may have values between zero and two times the momentum cut-off. Thus one has to extrapolate the dressing functions beyond the cut-off. This is accomplished by employing the ultraviolet asymptotic forms (B.12) and (B.13), which are given in appendix B.2. These forms are known to be analytic solutions of the continuum DSEs at large momenta. For typical torus sizes one does not exactly reproduce the one-loop scaling at large momenta, since the momentum cutoff is still rather low of the order of a few GeV. However, we need the extrapolation only for a small number of momentum configurations and our procedure therefore is satisfactory. We also checked other prescriptions, like setting the dressing functions to a constant value or even to zero for momenta larger than the cut-off and found only very minor changes in the solutions of the DSEs.

$V[\text{fm}^4]$	105964	51102	20931	4091	2461
$G(0)$	3.84	3.79	3.68	3.36	3.10

Table 2

Finite volume analysis: Dependence of $G(0)$ on the volume V at a fixed cutoff of $\Lambda = 2.47$ GeV.

3 Results

3.1 Finite volume effects at zero temperature

Our first application of the numerical tools described in the previous sections is to determine whether one can recover the solutions of the continuum DSEs, determined in ref. [10], also on a torus. First results of such an investigation have already been reported in refs. [10,25]. Here we extend this investigation to much larger volumes seeking for signals of the continuum limit on the torus. To clarify the effects in the most pronounced way we display results for the gluon *propagator*, $D(p^2) = Z(p^2)/p^2$, in fig. 3 and the ghost *dressing function*, $G(p^2)$, in fig. 4. The momentum scale on the torus is adapted to the one from the corresponding continuum calculation [17]. Our strategy was to fix the cutoff at a certain scale, $\Lambda = 2.47$ GeV for the solutions presented in figures 3 and 4, and then vary the numbers of points on the momentum lattice to generate different volumes of the torus. For p_0 in units of GeV and denoting the smallest momentum accessible on the (symmetric) lattice, the corresponding volume is determined by $V = (2\pi/p_0 \cdot 0.1973 \text{ fm})^4$. The two solutions displayed in figures 3 and 4 have been calculated with $V \approx 2500 \text{ fm}^4$ and $V \approx 10^5 \text{ fm}^4$. Note that the smaller volume corresponds to a typical one achieved in recent lattice simulations [13], whereas the larger value exceeds the current possibilities of the lattice formulation by far. Interestingly, our results for these two volumes are very similar to each other, whereas there is a qualitative gap to the continuum results. The numerical continuum solution for the gluon propagator is in accordance with the infrared analytical result, eq. (B.10), and clearly shows a vanishing gluon propagator at zero momentum with the anomalous dimension $\kappa \approx 0.595$. The solutions on the torus, on the other hand, seem to go to a constant if extrapolated to zero momentum.

In order to analyse the zero momentum limit more precisely we fitted the infrared part of our solution to the form

$$\frac{Z_{ir}(p^2)}{p^2} = \frac{a_0}{p^2} \left(\frac{p^2}{a_1^2 + p^2} \right)^{2a_2}. \quad (24)$$

For momenta below $p \approx 250$ MeV we found very good agreement with our large volume solution for the values of the parameters $a_0 = 0.244$, $a_1 = 0.233$ GeV and $a_2 = 0.500$. Thus, to very good precision we find that the torus solution

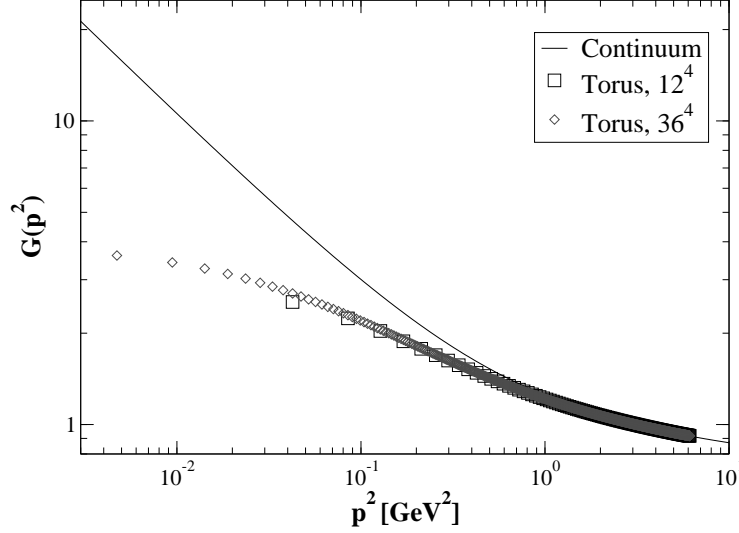


Fig. 4. The ghost dressing function $G(p^2)$ in the continuum and on the torus at two different volumes.

for the gluon propagator is constant in the infrared, and thus very different from its continuum limit. However, there is a caveat here. Finite size effects due to the ultraviolet momentum cutoff Λ_{UV} could introduce contributions of the order $1/\Lambda_{UV}$ or $1/\Lambda_{UV}^2$ to $D(0)$ on the torus. Provided these terms have large coefficients they could dominate even for very small momenta and hide a possible underlying power law with $\kappa > 0.5$. In order to test for finite size effects we solved the DSEs on a torus with fixed volume $V = 4091 \text{ fm}^4$ for five different cutoffs corresponding to momentum lattices of 16^4 , 20^4 , 24^4 , 28^4 and 32^4 . We then fitted the expression (24) in the infrared with a_2 fixed to 0.5. Our results for $D(0)$ are given in table 1. We find a considerable dependence of $D(0)$ on the cutoff, which can be described by the form

$$D(0) = d_0 + \frac{d_1}{\Lambda_{UV}} \quad (25)$$

with the parameters $d_0 = 2.67 \text{ GeV}^{-2}$ and $d_1 = 5.57 \text{ GeV}^{-1}$. If we use Λ_{UV}^2 instead of Λ_{UV} the quality of the fit gets much worse, indicating that the dominant finite size effects are of order $1/\Lambda_{UV}$. Nevertheless, the nonvanishing, cutoff independent constant d_0 indicates that the gluon propagator on a torus is indeed finite in the infrared.

Our solutions for the ghost dressing function on a torus are shown in fig. 4. Similar to the gluon we note a clear qualitative difference between the torus and the continuum solutions. Although there is a distinct increase of $G(p^2)$ in the midmomentum region, the function is much less steep in the infrared than the continuum solution. The volume dependence for a given cutoff is very small and it is not at all clear that $G(0) \rightarrow \infty$ for $V \rightarrow \infty$. On the contrary, the infrared part of the ghost dressing function for the large volume can be

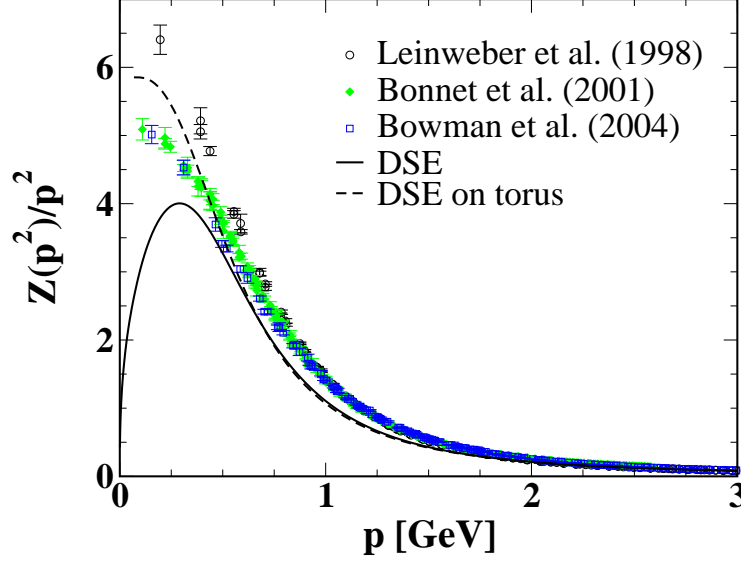


Fig. 5. The gluon propagator $Z(p^2)/p^2$ in the continuum and on the torus compared to the lattice results of refs. [13,26,27]. The most recent lattice data from ref. [28] agree well with the data shown.

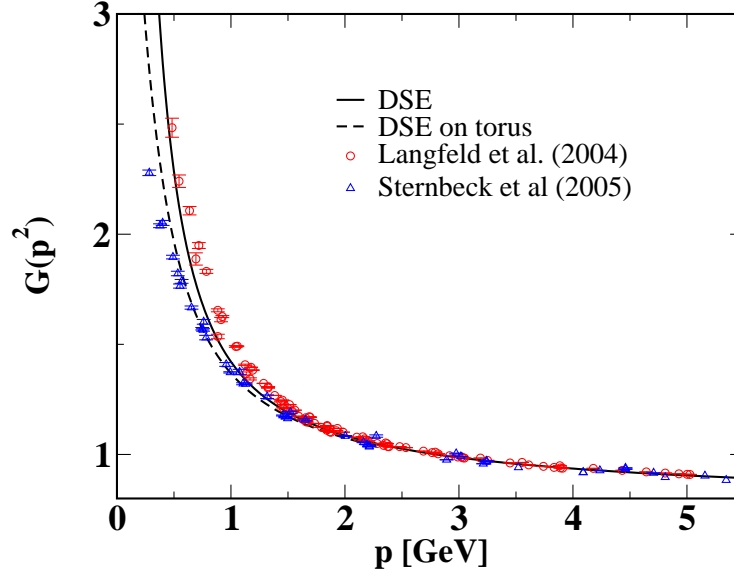


Fig. 6. The ghost dressing function $G(p^2)$ in the continuum and on the torus compared to the lattice results of refs. [28,29].

fitted very well with the form

$$G_{ir}(p^2) = \frac{e_0}{(e_1 + p^2)^{e_2}}, \quad (26)$$

where $e_0 = 0.11\text{GeV}^{2e_2}$, $e_1 = 0.02\text{GeV}^2$ and $e_2 = 0.32$. If the scale e_1 is not chosen as a fit parameter but set to zero the fit becomes unstable. This clearly indicates that the dressing function is finite when extrapolated to zero momentum. From this fit form we can determine $G(0)$ for several volumes.

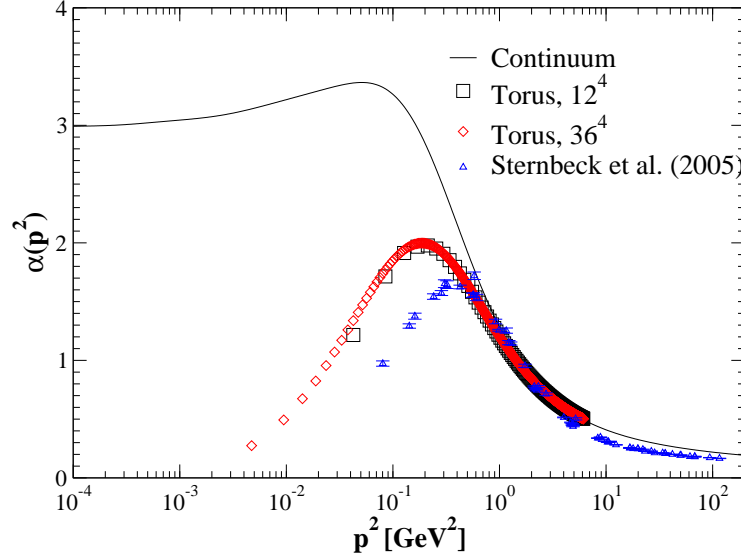


Fig. 7. The running coupling in the continuum and on the torus compared to the lattice results of ref. [28].

Our results are given in table 2. From these values it is not clear whether the ghost diverges in the infinite volume limit. If so, then this limit is approached at an extremely slow rate. A possible polynomial representation of the values in table 2 is given by $G(0) = g_0 - g_1/V^{g_2}$ with $g_0 = 3.89$, $g_1 = 0.305$ and $g_2 = 0.668$, which indeed indicates a constant ghost in the infinite volume limit. However, other forms may fit the values equally well.

In figs. 5 and 6 we compare our results for the gluon propagator and the ghost dressing function with corresponding ones from recent lattice calculations [13,26,27,28,29] (see also [30] for first results on large asymmetric lattices). Interestingly enough, there is a striking qualitative agreement between the lattice data and our torus result for the gluon propagator. Both seem to go to a finite value in the infrared, in disagreement with the continuum solution. Thus, provided the analogy between the lattice formulation and the formulation of the DSEs on the torus holds, then the 'true', continuum limit gluon propagator may look quite different in the infrared than the one of contemporary lattice calculations. As for the ghost, both results, the DSE solution in the continuum and the one on the torus look strikingly similar on a linear plot of the momentum range assessible on the lattice. In turn, this means that the lattice ghost propagator could equally well be divergent or finite at zero momentum.

The running coupling (10), determined from a combination of the ghost and gluon dressing functions, is shown in fig. 7. Whereas in the continuum the coupling approaches the infrared fixed point (B.11) for small momenta, we find an infrared vanishing coupling on the torus. Note that the difference between the small volume (12^4) and large volume (36^4) result on the torus is

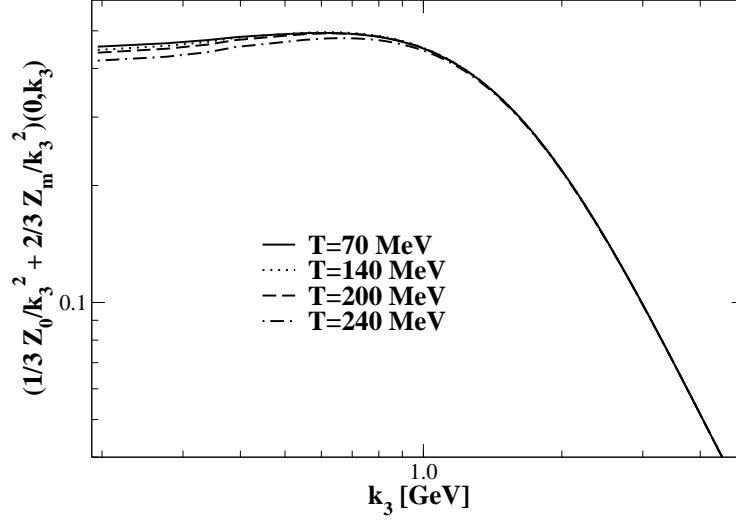


Fig. 8. The zeroth Matsubara component of the gluon propagator, Z_T/k_3^2 , in the infrared region at different temperatures (24^3 momentum grid).

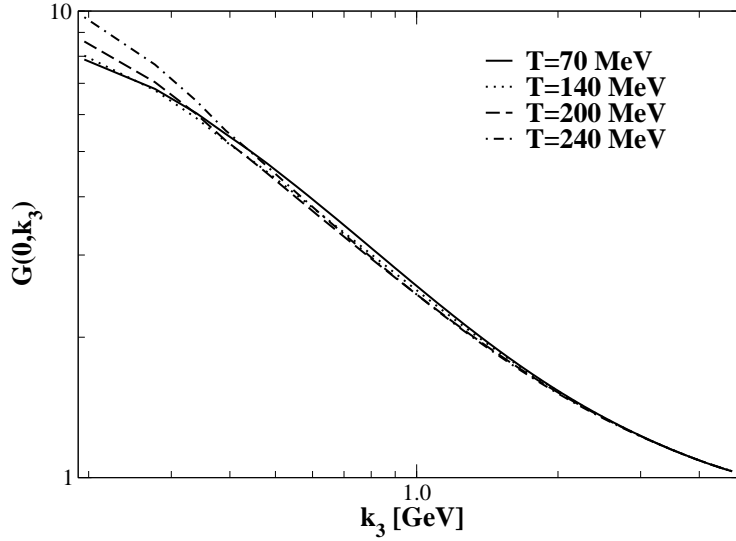


Fig. 9. The ghost dressing function G in the infrared region at different temperatures (24^3 momentum grid).

very small. Thus one cannot extrapolate to the continuum limit by varying the torus volume. Compared to the results of recent lattice calculations [28] we again find qualitative agreement between the torus solutions and the lattice data¹.

3.2 Ghost and gluon propagators at temperatures below T_c

A convenient way to assess the temperature dependence of the gluon dressing functions is to evaluate the linear combination $Z_T(k)$, eq. (14), and the difference of the dressing functions Z_0 and Z_m ,

$$\Delta Z(k) = Z_0(k) - Z_m(k). \quad (27)$$

The linear combination (14) connects directly to the gluon dressing function $Z(p^2)$ at zero temperature whereas ΔZ measures temperature effects on the tensor structure of the gluon propagator.

The obtained results have in common that the ghost and gluon propagators are only slightly temperature dependent for $T \leq 240\text{MeV}$. Hereby the most interesting parts are their zeroth Matsubara modes since all other modes turn out to be $O(4)$ invariant, *i.e.* they depend to a very high precision only on $p^2 = \vec{p}^2 + p_4^2$, and therefore show no explicit temperature effects.

The results presented in figs. 8 and 9 have been calculated using a 24^3 grid and an ultraviolet cutoff of 4.7 GeV. This corresponds to a spatial volume of approximately 6 fm^3 . In figs. 8 and 9 we show the zeroth Matsubara modes of the ghost dressing function G and the quantity Z_T/k_3^2 at several temperatures. As has been discussed in the previous section only the finite-volume value of the corresponding infrared exponents can be extracted. The propagator function Z_T/k_3^2 shows only small variations for a large range of temperatures up to at least 240 MeV. The ghost dressing functions also changes only by a few percent. The quantity ΔZ (27) which measures the change of the tensor structure of the gluon propagator is displayed in fig. 10. Whereas ΔZ remains quite small for small temperatures, it increases drastically in the mid-momentum regime around $k_3 = 700\text{ MeV}$ when the temperature is raised above 200 MeV, thereby indicating a sizeable change of the gluon propagator in this region. These temperature effects are presumably most important just below the critical temperature. Thus the question arises whether higher Matsubara modes are important for this quantity. As one sees from fig. 11 this is definitely not the case for $T=140\text{ MeV}$. At larger temperatures we find even smaller contributions from these modes.

At $T=140\text{ MeV}$ we have also studied the system for considerably larger volumes. As one can see from figs. 12 and 13 the changes when going to larger volumes are, even in the infrared, minute. Of course, this has to be expected from the results presented in previous subsection. Nevertheless, as discussed this does not mean that the infinite-volume limit of these quantities is reached.

¹ The quantitative difference between the two approaches at large momenta is due to missing two-loop contributions in the gluon-DSE.

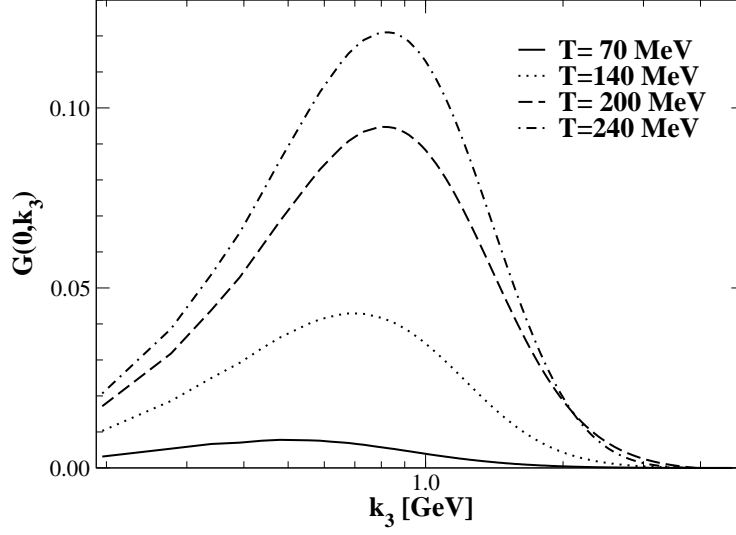


Fig. 10. The difference ΔZ of the gluon dressing functions at different temperatures (24^3 momentum grid).

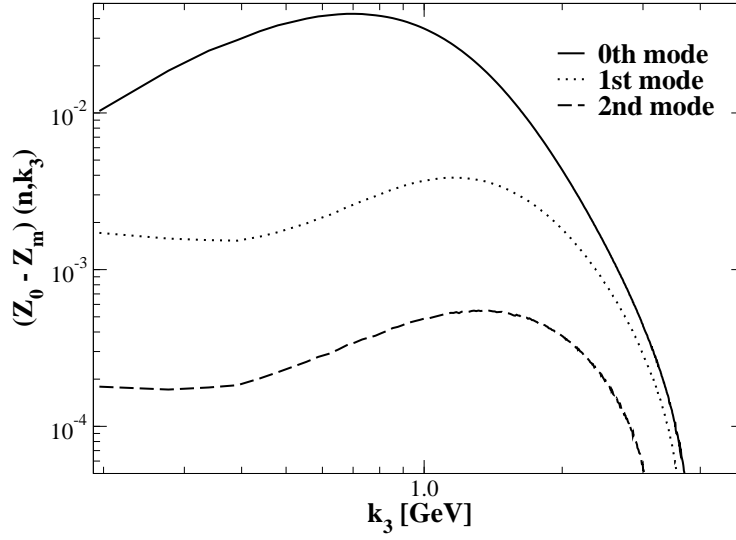


Fig. 11. Different Matsubara modes of the difference ΔZ of the gluon dressing functions at $T = 140$ MeV (24^3 momentum grid).

Finally, we want to mention that with this numerical method it is possible to find qualitatively similar solutions for the gluon and ghost dressing functions also for quite high temperatures up to $T=800$ MeV [23]. At the moment we lack a definite interpretation of such solutions, the most likely explanation is that they belong to a super-heating of the low-temperature phase.

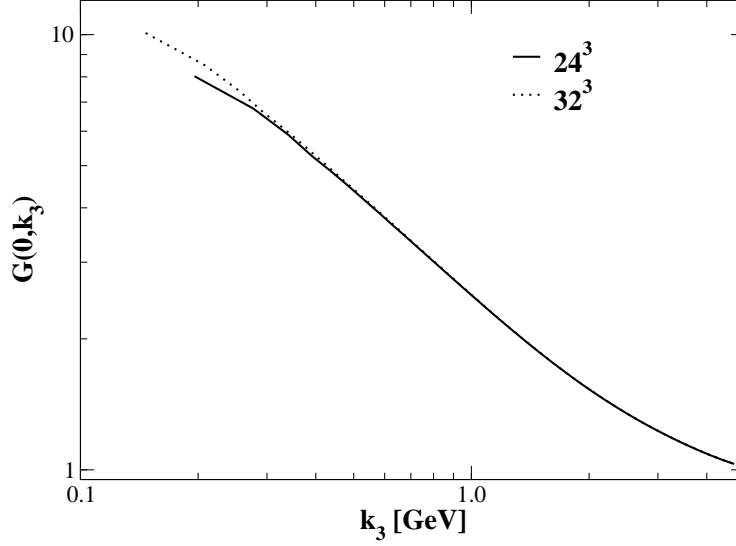


Fig. 12. The ghost dressing function G in the infrared region from different grid sizes at $T=140$ MeV).

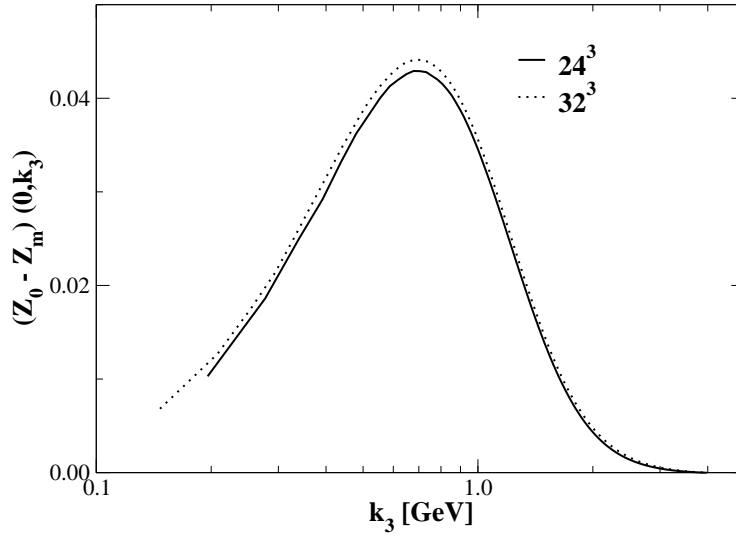


Fig. 13. The difference ΔZ of the gluon dressing functions from different grid sizes at $T=140$ MeV

4 Summary

We presented solutions for the coupled set of Dyson-Schwinger equations for the ghost and gluon propagators of Landau gauge Yang-Mills theory on a four-dimensional torus. We studied the influence of the finite volume on the infrared behaviour of the propagators on a symmetric torus and we employed asymmetric tori to introduce non-vanishing temperatures.

Evaluating the ghost and gluon propagators on the torus for various volumes and momentum cutoffs provides a surprising result: Even for extremely

large volumes, exceeding contemporary lattice volumes by magnitudes, distinct qualitative differences between torus and continuum solutions are found. On the torus, the gluon propagator as well as the ghost dressing function are finite in the infrared, whereas the same quantities in the continuum limit are infrared vanishing (gluon) or diverging (ghost). By varying the ultraviolet cutoff we made sure that these effects are not artefacts of the ultraviolet regularization. Thus the continuum limit seems to be a nontrivial transition from the compact to the noncompact manifold.

We furthermore compared our results with contemporary lattice calculations. For the ghost dressing function all three solutions look similar in the momentum range accessible by lattice calculations. For the gluon propagator, however, we found qualitative agreement of the torus results with the lattice data in the infrared, whereas the continuum result is different. Thus our results suggest that the continuum gluon propagator may be qualitatively different than the one found in contemporary lattice calculations. The same is true for the running coupling. The analytical results from the continuum DSE-approach give a fixed point in the infrared. Although the value of this fixed point is, within narrow limits, subject to the employed ansatz for the ghost-gluon vertex [9], its very existence can be derived from the structure of the ghost-DSE alone and is therefore independent of a truncation [31]. The fixed point is universal [12] and shown to be invariant in a class of gauges that interpolate between Landau and Coulomb gauge [32]. On a compact manifold, however, both, the DSE-approach and lattice Monte-Carlo simulations [28,33,34,35] find a vanishing coupling in the infrared in disagreement with the analytically derived continuum limit.

A further problem studied in this work is the effect of non-vanishing temperatures on the infrared anomalous dimensions of the ghost and gluon propagators. Our results indicate that the anomalous dimensions are unaffected up to temperatures of the order of the phase transition. Moderate-size differences do occur in the tensor structures of the gluon propagator indicating a qualitative change in the gluon propagator as one approaches the phase transition from below. Nevertheless, it remains an open question which degrees of freedom are the most relevant for the deconfinement phase transition.

We presented a numerical method to solve coupled sets of nonlinear integral equations on a compact manifold by iteration. The method is suitable for both symmetric and asymmetric space-time tori. It can be applied to systems with an arbitrary number of dimensions and an arbitrary number of equations. We are confident that the algorithm presented here also provides a sound basis for investigations of related issues. *E.g.* the choice of twisted boundary conditions on the torus [36] might hold the key to provide a tool for assessing the influence of topologically non-trivial field configurations on the infrared behaviour of QCD Green's functions.

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A Integral kernels for the Dyson-Schwinger Equations

The two kernels of the ghost equation (18) are given by:

$$A_T(k, q) = -\frac{\vec{k}^2 \vec{q}^2 - (\vec{k} \cdot \vec{q})^2}{(\vec{k} - \vec{q})^2} \quad (\text{A.1})$$

$$A_L(k, q) = -\frac{k^2 q^2 - (kq)^2}{(k - q)^2} + \frac{\vec{k}^2 \vec{q}^2 - (\vec{k} \cdot \vec{q})^2}{(\vec{k} - \vec{q})^2}. \quad (\text{A.2})$$

The kernel of the ghost loop and the four kernels of the gluon loop for the heat-bath transversal part of the gluon equation (19) are:

$$R(k, q) = -\frac{(\vec{q}^2 \vec{k}^2 - (\vec{k} \cdot \vec{q})^2)}{\vec{k}^2} \quad (\text{A.3})$$

$$M_T(k, q) = -2 \frac{\vec{q}^2 \vec{k}^2 - (\vec{k} \cdot \vec{q})^2}{\vec{k}^2 \vec{q}^2 \vec{p}^2} \left((\vec{k} \cdot \vec{q})^2 + \vec{k}^2 \vec{q}^2 + 2\vec{p}^2 (\vec{k}^2 + \vec{q}^2) \right) \quad (\text{A.4})$$

$$M_1(k, q) = -2 \frac{(q_0 \vec{k} \cdot \vec{q} - k_0 \vec{q}^2)^2 \left((\vec{k} \cdot \vec{p})^2 + \vec{k}^2 \vec{p}^2 \right)}{\vec{k}^2 \vec{q}^2 \vec{p}^2 q^2} \quad (\text{A.5})$$

$$M_2(k, q) = -2 \frac{(p_0 \vec{k} \cdot \vec{p} - k_0 \vec{p}^2)^2 \left((\vec{k} \cdot \vec{q})^2 + \vec{k}^2 \vec{q}^2 \right)}{\vec{k}^2 \vec{q}^2 \vec{p}^2 p^2} \quad (\text{A.6})$$

$$M_L(k, q) = -2 \frac{(\vec{q}^2 \vec{k}^2 - (\vec{k} \cdot \vec{q})^2) (q^2 p^2 - q_0 p_0 q p)^2}{\vec{k}^2 \vec{q}^2 \vec{p}^2 k^2 p^2}. \quad (\text{A.7})$$

The kernels for the heat-bath longitudinal part (20) have the form:

$$P(k, q) = -\frac{(q_0 \vec{k}^2 - k_0 \vec{k} \cdot \vec{q})^2}{k^2 \vec{k}^2} \quad (\text{A.8})$$

$$N_T(k, q) = -2 \frac{(k_0 \vec{k} \cdot \vec{q} - q_0 \vec{k}^2)^2 ((\vec{q} \cdot \vec{p})^2 + \vec{q}^2 \vec{p}^2)}{\vec{k}^2 \vec{q}^2 \vec{p}^2 k^2} \quad (\text{A.9})$$

$$N_1(k, q) = -2 \frac{(\vec{q}^2 \vec{k}^2 - (\vec{k} \cdot \vec{q})^2) (k^2 q^2 - k_0 q_0 k q)^2}{\vec{k}^2 \vec{q}^2 \vec{p}^2 k^2 p^2} \quad (\text{A.10})$$

$$N_2(k, q) = -2 \frac{(\vec{q}^2 \vec{k}^2 - (\vec{k} \cdot \vec{q})^2) (k^2 p^2 - k_0 p_0 k p)^2}{\vec{k}^2 \vec{q}^2 \vec{p}^2 k^2 q^2} \quad (\text{A.11})$$

$$N_L(k, q) = -\frac{1}{2 \vec{k}^2 \vec{q}^2 \vec{p}^2 k^2 q^2 p^2} \left[(\vec{p} \cdot \vec{k}) \vec{q}^2 (k_0 p^2 + p_0 k^2) - (\vec{p} \cdot \vec{q}) \vec{k}^2 (p_0 q^2 + q_0 p^2) + (\vec{k} \cdot \vec{q}) \vec{p}^2 (k_0 q^2 - q_0 k^2) \right]^2. \quad (\text{A.12})$$

B Numerical method

B.1 The algorithm

The numerical method is based on approximating the unknown exact solution of the coupled system of integral equations by an iterative procedure. Fixed-point iteration works fine enough for simple cases, but in the present example suffers from extremely poor convergence. We therefore employed the Newton-Raphson method [37], which improves the convergence drastically, see *e.g.* ref. [7] (where this method has been employed for the first time for DSEs in the continuum) and ref. [16]. In the symbolic notation of eq. (21) we can write the DSEs as

$$f(D_i) = \frac{1}{D_i} - \frac{1}{D(\mu^2)} - \Pi_D(k_i^2) + \Pi_D(\mu^2) = 0, \quad (\text{B.1})$$

with $D_i = D(k_i)$. The Newton-Raphson method then relies on linearly improved iteration steps

$$D_i^{\eta+1} = D_i^\eta - \delta D_i^{\eta+1}, \quad \delta D_i^{\eta+1} = J^{-1}(D_i^\eta) f(D_i^\eta) \quad (\text{B.2})$$

with

$$J(D_i^\eta) = \frac{\partial f(D_i^\eta)}{\partial D_i^\eta}. \quad (\text{B.3})$$

We employ the following convergence criterion:

$$\left| \frac{\delta D_i^{\eta+1}}{D_i^\eta} \right| < \varepsilon \quad \text{for } i \in 1 \dots N. \quad (\text{B.4})$$

Note that one should not use a convergence criterion for an averaged relative change in the variables as the dressing functions $D(k_i^2)$ are rather steep in the infrared, where only a few points k_i are evaluated. Thus one can have situations where the majority of variables D_i with k_i in the ultraviolet are converged, whereas those D_i which correspond to small k_i still change drastically.

Furthermore, we observed that the algorithm converges much better when the Newton step is only performed partly when one is still far away from the exact solution. The weight ω_{Newton} is calculated as

$$\omega_{\text{Newton}} = 0.99 \min \left\{ \left| \frac{D_i^\eta}{\delta D_i^{\eta+1}} \right|, \quad i \in \{1, \dots, N\} \right\}. \quad (\text{B.5})$$

If $\omega_{\text{Newton}} > 1$ it is set to one. This condition also ensures that the dressing functions are positive definite on all intermediate iteration steps.

We determine the inverse of the matrix $J(D_i^\eta)$ approximately by employing a generalisation of the Gauss-Seidel method called 'successive overrelaxation' (SOR). This procedure is much faster than the exact inversion of the matrix. An exact determination of $J(D_i^\eta)^{-1}$ is not needed anyway, since the inverse determines just the next respective iteration step, but is not used in calculating the right hand side of the integral equations.

The SOR-algorithm necessarily converges, if the matrix J is strictly diagonally dominated, *i.e.*

$$|J_{ii}| \geq \epsilon + \sum_{\substack{j=1 \\ j \neq i}}^N |J_{ij}|. \quad (\text{B.6})$$

The algorithm starts with an initial guess for the solution of the linear equation $Jx = b$ with $x, b \in \mathbb{R}^N$ and $J \in \mathbb{R}^{N \times N}$. This guess is improved at each SOR-step $\eta \rightarrow \eta + 1$ according to:

$$x_i^{\eta+1} = (1 - \omega)x_i^\eta + \frac{\omega}{J_{ii}} \left(b_i - \sum_{j=1}^{i-1} J_{ij}x_j^{\eta+1} - \sum_{j=i+1}^N a_{ij}x_j^\eta \right). \quad (\text{B.7})$$

The relaxation parameter ω interpolates linearly between the old vector x^η and the Gauss-Seidel rule $\omega/J_{ii}(\dots)$. The algorithm converges for values of ω in the range $0 < \omega < 2$. If the matrix J is strictly diagonally dominant, then convergence is best for $1 < \omega < 2$ (*i.e.* 'overrelaxation'). However, other cases may give best results for a value between $0 < \omega < 1$ ('underrelaxation').

We tried values for ω between 0.7 and 1.2 and obtained best results for $w \approx 0.8$. This clearly indicates, that the Jacobian matrix is not strictly diagonally dominant but still well behaved enough for the SOR algorithm to converge. For the update of the i -th component of x , its relative change is

$$\delta_i^{\eta+1} = \frac{\omega}{b_i} \left(b_i - \sum_{j=1}^{i-1} J_{ij} x_j^{\eta+1} - \sum_{j=i}^N J_{ij} x_j^{\eta} \right). \quad (\text{B.8})$$

These quantities are used to calculate an estimated averaged error from the sum of the absolute values of the relative changes in the current approximation to the solution

$$\varepsilon^{\eta+1} = \frac{1}{N} \sum_{i=1}^N |\delta_i^{\eta+1}|. \quad (\text{B.9})$$

The criterion for convergence is, that $\varepsilon^{\eta+1} < \epsilon$, for a given small ϵ .

In our implementation of the Newton-Raphson method the SOR-algorithm has to be called at every step of the Newton-iteration. In general it delivers a converged result for J^{-1} . In the rare occasions where the SOR-algorithm did not converge, we determined J^{-1} with a Gauss-Jordan elimination and returned to the SOR-algorithm in the next Newton-step.

B.2 Start guess for the dressing functions

The Newton-Raphson iteration method described in the last section is a local method, which converges quadratically, if the start guess for the dressing functions is close enough to the final solutions. The exact meaning of 'close enough', however, depends very much on the problem at hand. If the dressing functions are infrared finite, as for example in the Dyson-Schwinger equations for the quark propagator [38,39], then convergence is achieved for almost any form of the start guess. However, if the dressing functions are infrared singular, as in the present case, much more care is required and one should provide a start guess that is qualitatively similar to the actual solution². In our case such a start guess can be provided by noting that the continuum version of the DSEs (18),(19) and (20) can be solved analytically in the zero temperature limit for both very large and very small momenta.

In the infrared asymptotic limit, the ghost and gluon dressing functions, $G(p^2)$ and $Z(p^2)$, are proportional to simple power laws [6]

$$Z(p^2) \sim (p^2)^{2\kappa}, \quad G(p^2) \sim (p^2)^{-\kappa}, \quad (\text{B.10})$$

² As an alternative one can resort to a globally converging method, see ref. [16].

Parameters	C	D_s	β	ν	η
$G(p^2)$	0.35	0.02	-0.3	0.5	-0.2
$Z_m(p^2), Z_0(p^2)$	550	0.01	1.2	2.0	-0.6

Table B.1

Parameters for the start guesses for the gluon and ghost dressing functions.

with $\kappa = \frac{93 - \sqrt{1201}}{98} \approx 0.595353$ [5,9]. Correspondingly the running coupling (10) has an infrared fixed point,

$$\alpha(0) = \frac{4\pi}{6N_c} \frac{\Gamma(3-2\kappa)\Gamma(3+\kappa)\Gamma(1+\kappa)}{\Gamma^2(2-\kappa)\Gamma(2\kappa)} \approx \frac{8.92}{N_c}. \quad (\text{B.11})$$

Both, the values of the exponent κ and the fixed point $\alpha(0)$ have recently been shown to be independent of the gauge parameter in a class of gauges that interpolate between Landau and Coulomb gauge [32].

In the ultraviolet asymptotic limit one recovers the well known expressions from resummed perturbation theory,

$$Z(p^2) = Z(\mu^2) \left[\frac{11N_c\alpha(\mu^2)}{12\pi} \log \left(\frac{p^2}{\mu^2} \right) + 1 \right]^\gamma, \quad (\text{B.12})$$

$$G(p^2) = G(\mu^2) \left[\frac{11N_c\alpha(\mu^2)}{12\pi} \log \left(\frac{p^2}{\mu^2} \right) + 1 \right]^\delta. \quad (\text{B.13})$$

Here $Z(\mu^2)$ and $G(\mu^2)$ denote the value of the dressing functions at some renormalisation point μ^2 and γ and δ are the respective anomalous dimensions. To one loop order one has $\delta = -9/44$ and $\gamma = -1 - 2\delta = -13/22$ for arbitrary number of colours N_c and no quarks, $N_f = 0$.

A simple ansatz that interpolates between both limits is given by

$$F(p^2) = C \left(\frac{p^2}{p^2/D_s + 1} \right)^\beta \left(\nu \log(p^2/D_s + 1) + 1 \right)^\eta, \quad (\text{B.14})$$

where C, D_s, β, ν, η are parameters to be chosen appropriately. Our choice of parameters for the ghost dressing function and the heat bath transverse and longitudinal dressing of the gluon are listed in table 3. As long as the qualitative properties of the start guess remains unchanged for the relevant momenta convergence is achieved for a whole range of parameter choices centred around the values given.

References

- [1] L. von Smekal and R. Alkofer, Proceedings of the 4th International Conference on Quark Confinement and the Hadron Spectrum (CONFINEMENT IV), arXiv:hep-ph/0009219.
- [2] A. Hauck, L. von Smekal and R. Alkofer, Comput. Phys. Commun. **112**, 166 (1998) [arXiv:hep-ph/9804376].
- [3] N. Nakanishi and I. Ojima, “Covariant Operator Formalism Of Gauge Theories And Quantum Gravity,” World Sci. Lect. Notes Phys. **27**, 1 (1990).
- [4] T. Kugo and I. Ojima, Prog. Theor. Phys. Suppl. **66**, 1 (1979).
- [5] D. Zwanziger, Phys. Rev. D **65**, 094039 (2002).
- [6] L. von Smekal, R. Alkofer and A. Hauck, Phys. Rev. Lett. **79** (1997) 3591 [arXiv:hep-ph/9705242];
L. von Smekal, A. Hauck and R. Alkofer, Annals Phys. **267** (1998) 1 [arXiv:hep-ph/9707327].
- [7] D. Atkinson and J. C. R. Bloch, Phys. Rev. D **58**, 094036 (1998) [arXiv:hep-ph/9712459].
- [8] R. Alkofer and L. von Smekal, Phys. Rept. **353** (2001) 281 [arXiv:hep-ph/0007355].
- [9] C. Lerche and L. von Smekal, Phys. Rev. D **65**, 125006 (2002) [arXiv:hep-ph/0202194].
- [10] C. S. Fischer and R. Alkofer, Phys. Lett. B **536**, 177 (2002) [arXiv:hep-ph/0202202].
- [11] J. M. Pawłowski, D. F. Litim, S. Nedelko and L. von Smekal, Phys. Rev. Lett. **93**, 152002 (2004) [arXiv:hep-th/0312324]. C. S. Fischer and H. Gies, JHEP **0410**, 048 (2004) [arXiv:hep-ph/0408089].
- [12] R. Alkofer, C. S. Fischer and F. J. Llanes-Estrada, Phys. Lett. B **611**, 279 (2005) [arXiv:hep-th/0412330].
- [13] F. D. Bonnet *et al.*, Phys. Rev. D **64**, 034501 (2001).
- [14] P. J. Silva and O. Oliveira, Nucl. Phys. B **690**, 177 (2004) [arXiv:hep-lat/0403026].
- [15] C. S. Fischer, PhD thesis, Tübingen 2003, arXiv:hep-ph/0304233.
- [16] A. Maas, arXiv:hep-ph/0504110.
- [17] C. S. Fischer, R. Alkofer and H. Reinhardt, Phys. Rev. D **65** (2002) 094008 [arXiv:hep-ph/0202195].
- [18] J. I. Kapusta, *Finite-temperature field theory* (Cambridge University Press, Cambridge, 1993).
- [19] M. L. Bellac, *Thermal Field Theory* (Cambridge University Press, Cambridge, 1996).
- [20] A. Cucchieri, T. Mendes and A. Mihara, JHEP **0412** (2004) 012 [arXiv:hep-lat/0408034].

- [21] W. Schleifenbaum, A. Maas, J. Wambach and R. Alkofer, Phys. Rev. D **72** (2005) 014017 [arXiv:hep-ph/0411052].
- [22] A. Maas, J. Wambach, B. Gruter and R. Alkofer, Eur. Phys. J. C **37**, 335 (2004) [arXiv:hep-ph/0408074].
- [23] B. Gruter, R. Alkofer, A. Maas and J. Wambach, Eur. Phys. J. C **42** (2005) 109 [arXiv:hep-ph/0408282].
- [24] A. Maas, J. Wambach and R. Alkofer, Eur. Phys. J. C **42** (2005) 93 [arXiv:hep-ph/0504019].
- [25] C. S. Fischer, F. Llanes-Estrada and R. Alkofer, Nucl. Phys. Proc. Suppl. **141** (2005) 128 [arXiv:hep-ph/0407294].
- [26] D. B. Leinweber, J. I. Skullerud, A. G. Williams and C. Parrinello [UKQCD collaboration], Phys. Rev. D **58** (1998) 031501 [arXiv:hep-lat/9803015].
- [27] P. O. Bowman *et al.*, Phys. Rev. D **70**, 034509 (2004) [arXiv:hep-lat/0402032].
- [28] A. Sternbeck, E. M. Ilgenfritz, M. Mueller-Preussker and A. Schiller, Phys. Rev. D **72**, 014507 (2005) [arXiv:hep-lat/0506007].
- [29] J. Gattnar, K. Langfeld and H. Reinhardt, Phys. Rev. Lett. **93**, 061601 (2004) [arXiv:hep-lat/0403011].
- [30] O. Oliveira and P. J. Silva, AIP Conf. Proc. **756**, 290 (2005) [arXiv:hep-lat/0410048].
- [31] P. Watson and R. Alkofer, Phys. Rev. Lett. **86** (2001) 5239 [arXiv:hep-ph/0102332].
- [32] C. S. Fischer and D. Zwanziger, Phys. Rev. D **72**, 054005 (2005) [arXiv:hep-ph/0504244].
- [33] S. Furui and H. Nakajima, Phys. Rev. D **69** (2004) 074505 [arXiv:hep-lat/0305010].
- [34] B. Alles *et al.*, Nucl. Phys. B **502** (1997) 325.
- [35] P. Boucaud *et al.*, JHEP **0304**, 005 (2003) [arXiv:hep-ph/0212192]; P. Boucaud *et al.*, arXiv:hep-ph/0505150.
- [36] T. Tok, K. Langfeld, H. Reinhardt and L. von Smekal, “The gluon propagator in lattice Landau gauge with twisted boundary conditions”, to appear in the proceedings of Lattice05, Dublin, 2005. arXiv:hep-lat/0509134.
- [37] W. H. Press, S. A. Teukolsky, and W. T. Vetterling, B. P. Flannery, *Numerical Recipes in C*, Cambridge University Press, Cambridge, UK, 1992.
- [38] P. Maris and C. D. Roberts, Int. J. Mod. Phys. E **12**, 297 (2003) [arXiv:nucl-th/0301049].
- [39] C. S. Fischer and R. Alkofer, Phys. Rev. D **67** (2003) 094020 [arXiv:hep-ph/0301094].